sight affect my brain, my muscles. A whole series of physical effects is found in every case of the happening of sensory responses. A popular account would be content to say that we go direct from smoke to inferring fire. A more careful one will say that we go from an intra-organic event to smoke and from that to fire. But this is the difference between a more careful and extensive account and a rougher, more summary account. It is a difference in the detail of a series continuously physical in all its constituents. At no point is there a switch from one order or genus of Being to another. And without such a switch there is neither epistemological dualism nor does the demand for an epistemological monism arise. The key to the notion that there is such a switch (to be formulated in a dualism or explained away in a monism) arises from failing to note that representation is an *evidential function* which supervenes upon an occurrence, and from treating it as an inherent part of the structure of the organic events found in sensings. There are no physical events which contain representation of other events as part of their structure. Hence a separate world called psychical is provided for these hasty products of elision and telescoping.

To avoid misunderstanding let me say that the retort that the smoke is not a "conscious datum" while sensations and cerebral events are conscious data is not a reply, but a repetition of the same ignoring of the position. For the position herein recapitulated holds that to call anything "conscious" (so far as the requirements of this argument concern the word) is simply to say that it figures within the inferential or evidential function.

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REVIEWS AND ABSTRACTS OF LITERATURE

Wissenschaftslehre. B. BOLZANO. *Hauptwerke der Philosophie in original getreuen Neudrucken.* Band VII. Neu herausgegeben von Alois Häfler. Leipzig: Verlag von Felix Meiner, 1914. Zweiter Band. Pp. viii + 568.

In Volume II. of the *Wissenschaftslehre,* Bolzano continues[1] his exposition of the "elementary theory" of which the first part was considered in Volume I. The second part Bolzano devotes to general qualities of propositions and classification of propositions. This classification is made first, with reference to properties (Vol. I., § 80) and second, with reference to relations. A final chapter of the second part contains an exposition of propositions containing terms having

[1] The first volume of the *Wissenschaftslehre* was reviewed in this JOURNAL, Vol. XIII., p. 328.

a technical logical signification. An elaborate and valuable historical account of the subject-matter of the second part is added. The third part deals with true propositions. The fourth part has to do with inferences; this part is also accompanied by an historical appendix.

The second part of the elementary theory, to which more than one half of the volume is devoted, treats of propositions in themselves following a scheme closely resembling that which underlies Bolzano's discussion of representations (ideas) in the first part. In sections 123–129 the proposition as a complex concept is considered; these sections may be roughly correlated with sections 38, 54, 55, 57, 76, 79, 81 of Russell's *Principles of Mathematics.* Corresponding to the domain of a representation, the domain of a proposition is introduced, as well as its extension (§§ 130, 131, p. 25). Bolzano assumes that the domain of a proposition is the same as the domain of its subject-representation (*cf.* §§ 66, 151–153). All true propositions have at least one object in their domain; there are propositions with an empty domain (§ 146), but these are *false* (§§ 137, 138, 154, 4, § 160, note). Thus the propositions "round squares are virtuous" and "round squares are not virtuous" are both false. In making this assumption Bolzano is confronted with the task of reconciling the falsity of propositions of the preceding type with the truth of propositions such as "round squares do not exist." This problem (pp. 329, 330, 404) Bolzano attempts to solve by recourse to a theory of "representation of a representation" (*cf.* symbolic representations, § 90). Underlying this problem are such logical subtleties as the truth of falsity,[2] the validity of non-validity,[3] the extension of a class which has no extension,[4] the agreement of contradictoriness,[5] the predicability of non-predication,[6] denoting concept which denotes nothing,[4] *etc.* In this connection Bolzano has rather inadequately discussed "nothing" (§ 170). The convention which Bolzano has adopted in regard to the proposition with an empty domain is opposed to the assumption of certain modern logicians,[6] while others[7] take a mediating attitude. The problem is, of course, one of modern interest[8] whose definite solution has not yet been found. A remarkable feature of the problem is its relativity. Bolzano's specific con-

[2] *Cf.* Plato, *Theætetus,* 189, Sophist, 237.

[3] *Cf.* A. Liebert, *Kant Studien,* Ergänzungshefte, No. 32, pp. 8, 143.

[4] *Cf.* Russell, *Principles of Mathematics,* §§ 36, 41, 73, 78, 100, *etc.*

[5] *Cf.* Bolzano, *Wissenschaftslehre,* Vol. II., pp. 402, 403, 421–22, 435–37, 447, *etc.*

[6] See Russell, *Principles of Mathematics,* § 73; *Principia Mathematica,* Vol. I., p. 67.

[7] For example, Peirce, *American Journal of Mathematics,* Vol. VII., p. 187.

[8] See *The New Realism,* p. 485; *Kant Studien,* Ergänzungshefte No. 26, especially pp. 104–106, 131; James, *Psychology,* Vol. I., p. 463.

vention is formally desirable in the theory of the truth or falsity of an aggregate of propositions (§ 160). A doctrine of "empty" truth[9] is convenient in the foundations of geometry; in the theory of the working hypothesis a category of irrelevancy[10] is naturally suggested. It may be added that just as in Bolzano's theory the proposition "round squares do not exist" requires explanation, so the proposition "round squares exist" is a source of embarrassment to the uniform validity of the theory of "empty" truth.

In Chapter II. Bolzano defines simple and complex, conceptual and perceptual propositions (§§ 132–133; *cf.* § 221); propositions with representations of aggregates (§ 135) of negation (§ 136); propositions determining the domain of a representation (§§ 137–139; *cf.* §§ 173, 196); propositions expressing relations between representations (§ 140); existential (§ 142); problem setting (§§ 143, 145); and convertible propositions (§ 149). In sections 147–148 certain other properties of propositions are defined by means of a technique which I shall call Bolzano's *functional operation* on given representations, propositions, *etc.* Bolzano applies this operation rather systematically in his second volume. This operation is used also in the first volume; indeed, Bolzano states (p. 314): "In the sequel we shall frequently employ the variability of certain components of a representation and the derivation of new representations by the substitution of arbitrary representations for these components." On Bolzano's functional operation are based the validity of a proposition (§ 147), the distinction between analytic and synthetic propositions (§§ 148, 197), consistence and inconsistence of propositions (§ 154–159), the independence[11] of propositions (§160), the comparative validity or probability of a proposition (§ 161), the (implicative) relation between truths called "*Abfolge*" (§ 162) and the theories in sections 164–168 which are correlated with sections 154–162. In Chapter V. of the second part Bolzano's functional operation is referred to only indirectly, *e. g.*, sections 179, 180, which discuss propositions of the form "If . . . then . . ." and "A determines B."[12] Bolzano asserts (§ 154) that the most important rela-

[9] See, for example, E. V. Huntington, *Mathematische Annalen*, Vol. 73 (1912–13), pp. 549, 550; Huntington's term "inoperative" appears to have been suggested by my term "ineffective" (*Amer. Journal of Math.*, 1909, pp. 380, 381).

[10] *Cf.* Courant, *Journal für die reine und angewandte Mathematik*, Vol. 144, p. 190; A. R. Schweitzer, *Revue de Métaphysique et de Morale*, 1914, p. 176.

[11] The term "independence" which I have used above should be compared with Bolzano's term, pp. 144, 323, *etc.* Another mathematical sense of the term occurs, *e. g.*, in Grassmanns *Extensive Algebra* (*cf.* Bolzano, Vol. II., p. 227, 2), p. 238, Vol. I., 1 (Coll. Works).

[12] *Cf.* Russell, *Principles of Mathematics*, p. 383.

tions among propositions are those arising from the application of his functional operation. Incidentally, Bolzano has thoroughly anticipated Russell's "propositional function"; this is evident from § 147, p. 78, § 233, p. 427, § 223, p. 394, § 162, p. 192, *etc.* One of the most striking of Bolzano's anticipations and which, so far as I know, has not been noticed before, is presented by his theory of relations among an aggregate of propositions arising from their truth or falsity. Here Bolzano, in effect, gives a very complete account of the current theory of so-called independence of propositions in formal logic and mathematics; in fact Bolzano's theory of independence is far more comprehensive. In beginning his discussion Bolzano states (p. 163, *cf.* § 155 p. 113): "For the discovery of new truths it is of the highest importance to know of the truth or falsity of propositions in a certain aggregate and to ascertain how many of these propositions are true or false, the propositions having the expressions as presented, or the infinitely many expressions which these propositions may assume when certain of their parts are arbitrarily varied." This view has been brilliantly confirmed by the modern developments of mathematics, notably non-Euclidean geometry and the independent investigations of Hilbert in his celebrated *Foundations of Geometry*. Among the special cases (pp. 163–170) of the preceding problem, which I shall call "Bolzano's unconditioned problem of independence" Bolzano cites (pp. 163, 2; 167, 7) an aggregate of propositions giving rise to a variable number of true propositions including as extreme cases the verification and contradiction of the entire aggregate. To express this variability notationally, let P_1 $P_2, \ldots P_n$ be an aggregate of n propositions ($n = 2, 3, \ldots$) with representations $i, j, \ldots$ for which representations are arbitrarily substituted. Then let Σ be any set of n corresponding false propositions which thus arise, Σ_{i_1} ($i_1 = 1, 2, \ldots n$) be any set of derived propositions such that the propositions derived from P_{i_1} is true and the remaining derived propositions are false; $\Sigma_{i_1 i_2}$ (i_1 $i_2 = 1, 2, \ldots n, i_1 < i_2$) is any set of $[n(n-1)]/2.1$ derived propositions such that the propositions derived from P_{i_1}, P_{i_2} are true and the remaining derived propositions are false, and so on; finally $\Sigma_{12\ldots n}$ is any set of n derived propositions which correspond to P_1, $P_2, \ldots P_n$, and are all true. We have thus exhibited the range of Bolzano's variable relation based on the truth or falsity of propositions derived from a given aggregate by means of his functional operation; and, of course, for specific aggregates one or more of the sets $\Sigma, \Sigma_{i_1}, \Sigma_{i_1 i_2}, \Sigma_{i_1 i_2 i_3} \ldots$ may be "non-existent," *i. e.*, contain no propositions. In the specific example given by Bolzano (pp. 167, 170), namely, "Some X are A" and "Some X are not A" which we take as P_1 and P_2, respectively, the sets Σ, Σ_1, Σ_2, Σ_{12} are all

"existent" when X is variable, as Bolzano explicitly mentions. For according to Bolzano's assumption (discussed above) when a representation with no object is substituted for X, the propositions which arise have an empty domain and are false. Examples of a relation among propositions in mathematics in close analogy with the above have recently been given.[13] With the preceding unconditioned problem of independence Bolzano has correlated a conditioned problem (p. 167) as follows: "Especially remarkable and pertinent in the theory of relations of propositions is the restriction of the variability of representations i, j . . . by certain other propositions, in particular by requiring that only such representations be selected which *verify* certain other propositions." In connection with the preceding problems of independence it is noteworthy that Bolzano actually mentions (p. 166) an example from mathematics, namely, if A, B, C are three arbitrary non-collinear points in a Euclidean plane then (certain) two of the following relations are always verified:

$$\begin{array}{lll} \text{angle } A = \text{angle } B & \text{ang. } A < \text{ang. } B & \text{ang. } A > \text{ang. } B. \\ \text{segment } AC = \text{segment } BC & \text{seg. } AC > \text{seg. } BC & \text{seg. } AC < \text{seg. } BC. \end{array}$$

This example illustrates the following unconditioned type of independence according to Bolzano: "The number of true or false propositions in the aggregate M, N, O, . . . is known and this number is invariant under the substitution of arbitrary representations for the representations i, j, . . . in these propositions." I should say that the preceding example illustrates the conditioned correlative of this type. The modern notion of independence[14] in the foundations of geometry is easily subordinated under the latter conditioned type; here the invariant number, say I, of (properly) false propositions is unity and the range of variability of the set of associated representations i, j, . . . is finite, having a number, say N, equal to the number of propositions $P_1, P_2, \ldots P_n$ $(n > 1)$ presented.[15] Although Bolzano clearly sees the value of the problem of independence in discovery, he does not seem to recognize the practical use of this problem as an instrument in logical economy; tendencies in the latter direction are indicated by p. 164, 2, p. 123, 26, p. 165, 4 and elsewhere.

In section 96 (Volume I.) Bolzano defines the convertibility, *i. e.*, same validity, of representations. In section 156 he defines the same

[13] *Cf.* A. R. Schweitzer, *Bull. Amer. Math. Soc.*, Vol. 19, p. 70. E. V. Huntington, *ibid.*, Vol. 23, p. 276, where further references are given.

[14] *Cf.* Peano, *Rivista di Matematica*, Vol. IV., p. 62.

[15] Above type of independence admits obviously an extension to $I < n$, and $N = \dfrac{n(n-1)\cdots(n-I+1)}{I(I-1)\cdots\cdots 1}$.

validity or equivalence of propositions (*cf.* § 149). The validity of a proposition itself is defined (§ 147) thus: Consider the aggregate of different true propositions derived from a given proposition by the substitution (according to a certain rule) of representations for certain representations contained in the propositions; then the validity is the relation of this set of true propositions to the entire set derived as indicated. The corresponding degree of validity is a fraction with the limiting values 0, 1 (pp. 81, 82). The validity, then, determines the degree of probability of a given proposition under certain conditions. Evidently it is a relative concept, since for a given proposition it depends on the representations assumed to be variable. Analogous to the validity of a proposition is the comparative validity (§ 161) of a proposition M with reference to the propositions $A, B, C, D. \dots$ where in the latter the representations $i, j, \dots$ are considered variable and the propositions themselves are in the relation of consistence (§ 154) with regard to these variable representations. That is, Bolzano here defines the *probability* of a proposition M with reference to the propositions $A, B, C, D. \dots$ In section 162 a distinction between "external" and "internal" probability is made. As a magnitude, probability is a fraction with the limiting values 0, 1 (p. 172). Bolzano has made a valuable contribution to the logic of the classic theory of probability of Lacroix and Laplace (*cf.* p. 189) by his sharp formulations of fundamental concepts of this theory by means of propositions; this might be adopted with advantage in our modern treatises.[16] Bolzano's use of his functional operation in probability is paralleled by Couturat's application[17] of the "propositional function" to probability.

The third part of the elementary theory discusses exclusively true propositions and relations between these. The most remarkable of such relations Bolzano regards that of "*Abfolge*," in virtue of which certain truths are the reason (*Grund*) of others and the latter are in turn the consequence (*Folge*) of the former; in a technical sense, the relation is *dependence* (p. 323, § 221). The relation of "*Abfolge*" is mentioned in sections 162, 168 of the second part of the elementary theory; in particular in section 162, page 193. Bolzano applies to this relation the terms "material" and "formal." The formal "*Abfolge*" is a species of derivation (*Ableitbarkeit*). The concept "derivation" is discussed in sections 155–157, 162, 164. The relation of "*Abfolge*" is compared with derivation in sections 198, 200 and with causality in section 201. In sections 202, 203 Bolzano expresses the opinion that "*Abfolge*" is a simple concept; in section 221, page 388, he confesses that he sometimes suspects this relation to

16 See, for instance, A. A. Markoff, *Wahrscheinlichkeitsrechnung*, Leipzig, 1912.

17 *Encyclopædia of the Philosophical Sciences*, Vol. I., p. 154.

be complex and to be merely the concept of an order among truths such that from the smallest number of simple premises the largest possible number of remaining truths are derived as conclusions, *i. e.*, Bolzano suspects that "*Abfolge*" may be merged into derivation (*cf.* pp. 128–347). The relation of derivation holds between propositions "*ueberhaupt*" (§ 200) and anticipates, in essential respects, Russell's formal implication.[18] In sections 164, 179 (*cf.* § 177) Bolzano notes that the relation expressed by "if" and "then" is used to express derivation. It will be recalled that Russell[19] has observed that implication is involved in this relation. One form of Russell's material implication occurs incidentally in Bolzano's theory of derivation (p. 114); the other form, *viz.*, "implication between propositions not containing variables" (Russell, *loc. cit.*, p. 11) seems taken account of by Bolzano (§ 179) in the following words: "I do not believe that in every case in which an if and then is applied representations are regarded as variable . . . without disturbing the truth of the proposition . . . we frequently employ the if and then when no representations are regarded as variable in the compared propositions." Nevertheless, the example that Bolzano gives, "If x_0, x_1, are given integral numbers, $x_0=10\ \alpha_1+\alpha_2$, $x_1=10\ \alpha_2+\alpha_1$ then x_0-x_1 is divisible by 9" is not, at least according to Russell (*loc. cit.*, p. 6), an illustration of Bolzano's point. It is doubtful if Bolzano recognized the extreme[20] to which material implication can be carried. Bolzano would certainly disagree with Russell's assumption[21] that implication holds between any two propositions provided the first be false or the second true. In fact, Bolzano clearly indicates (p. 483) that he regards such statements as "The square is round implies Caius is immortal" as false.

The fourth part of the elementary theory concludes Volume II. of the *Wissenschaftslehre*. This part consists primarily of developments based on the relation of derivation (*cf.* p. 128). In section 226 Bolzano explains the nature of mathematical equality, anticipating, for instance, Grassmann,[22] and points out the forms of inference on which demonstrations of equality depend (pp. 407–409). Representations with no objects in their relation to inference are persistently mentioned, for example, in sections 225, 230, 234, 236, *etc.*; in particular, in sections 234-236, inferences grounded on the denial that representations have objects, are exhibited (*cf.* 55, note). In section 247 the systematic application of Bolzano's functional operation is resumed; sections 247–253 treat inferences drawn from propositions

18 *Principles of Mathematics*, pp. 14, 16, 33, 36, 106.

19 Russell, *loc. cit.*, Ch. III.

20 *Cf.* Bergmann, *Das philosophische Werk Bolzano's*, pp. 81, 89, 90.

21 *Cf.* J. B. Shaw, *Bull. Am. Math. Soc.*, Vol. 18, pp. 392–96.

22 *Collected Works*, Vol. I.–1, pp. 33, 34.

asserting relations between other propositions such as consistence, derivation, equivalence, probability, *etc.* These sections stand, of course, in close connection with corresponding sections of Part II. In dealing with inferences of probability (§ 253) Bolzano describes incomplete induction (p. 511) as distinguished from the complete (§ 236); the inference by analogy Bolzano regards as a particular case of incomplete induction (p. 512). Correspondingly, Bolzano indicates an extensive modification of logical processes on the basis of probability, such as improper and proper derivation, *etc.* This interesting section, diverging from Russell's views of induction, strongly supports Poincaré's remarks[23] on the latter subject. A remarkable feature of Parts III. and IV. is the use which Bolzano makes of his assumptions, "there exists a representation of widest domain" and "the number of simple concepts is finite." These assumptions occur in sections 221, 225, 226, 229, 233, 234, 236 (*cf.* pp. 150, 167 of Vol. II. and §§ 78, 99 of Vol. I.).

Volume II. of the *Wissenschaftslehre,* like Volume I., is most fascinating. Bolzano, indisputably, has made mistakes[24] in these volumes, but they are errors which, on the whole, might easily have been avoided by investigators of less aggressive originality. Both volumes offer excellent material for research.

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CHICAGO.

The Next Step in Democracy. R. W. SELLARS. New York: The Macmillan Company. 1916. Pp. 275.

Toward the great religious and social movements which have enlisted the hearts and minds of masses of men there have been two contrasting attitudes on the part of the educated and critical-minded minority. Some have been more impressed by the irrationality, the narrow dogmatism, the dangerous aspects which every such movement exhibits, and have opposed, scoffed, or ignored. Others have felt the appeal, seen the vision that it enshrined, and have sought, while identifying themselves with the movement and calling attention to its kernel of vital truth, to prune away its errors, purge it of its menace, and transform it into a force that should be wholly beneficial. This revisory and interpretative work subjects its exponents to denunciation from two sides—from the orthodox within the fold and from the skeptics without. They are called straddlers, hypocrites, heretics, traitors. But if they do succeed in leavening the movement with their saner view of its proper ends or means, in preserving its momentum and utilizing its power while making compar-

[23] *Science et Methode,* pp. 158–60, and Bolzano, W., II., p. 513, note.
[24] *Cf.* Bergmann, *loc. cit.,* pp. 66, 68, 79, 81, 89, 90, 94.